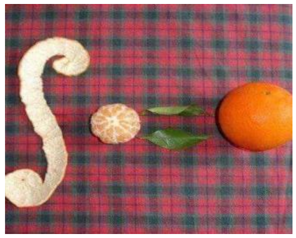
4- Integrals:



There are many who don't know how much important calculus is in our life and this why Isaac Newton invented this complementary part of mathematics because he had found that algebra and geometry didn't solve many mathematical problems. At that time mathematicians could calculate the velocity of ships for example, but failed to figure out the rate at which the ship was accelerating, so they needed a new mathematical method to solve the problems that involved changing variables; Newton spent about 18 months to form new theories in the science he called (the science of fluxions), which has evolved now for what we call (calculus) and which helped the engineer Apollo to chart the path between the earth and the moon .
In this days Medical Science Biologists use differential calculus to determine the exact rate of growth in a bacterial.

(4-1)Rules for Indefinite Integrals and Integration formulas for Basic Trigonometric Function:

DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x\,dx = -\cos x + C$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$

It is clear that this pattern does not fit the case of

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx$$

9- $\int \frac{1}{x} dx = \ln|x| + c$, $x \neq 0$

10- $\int e^x dx = e^x + c$

Example 1:

a) $$\int x^5\, dx = \frac{x^6}{6} + C$$

b) $$\int \frac{1}{\sqrt{x}}\, dx = \int x^{-1/2}\, dx = 2x^{1/2} + C = 2\sqrt{x} + C$$

c) $$\int \sin 2x\, dx = -\frac{\cos 2x}{2} + C$$

d) $$\int \cos \frac{x}{2}\, dx = \int \cos \frac{1}{2}x\, dx = \frac{\sin(1/2)x}{1/2} + C = 2 \sin \frac{x}{2} + C$$

$$\int [c_1 f_1(x) + c_2 f_2(x) + \cdots + c_n f_n(x)]\, dx = c_1 \int f_1(x)\, dx + c_2 \int f_2(x)\, dx + \cdots + c_n \int f_n(x)\, dx$$

Example 4

$$\int (3x^6 - 2x^2 + 7x + 1)\, dx = 3\int x^6\, dx - 2\int x^2\, dx + 7\int x\, dx + \int 1\, dx = \frac{3x^7}{7} - \frac{2x^3}{3} + \frac{7x^2}{2} + x + C$$

Example 5 Evaluate

(a) $\int \frac{\cos x}{\sin^2 x}\, dx$ (b) $\int \frac{t^2 - 2t^4}{t^4}\, dt$

Solution (a).

$$\int \frac{\cos x}{\sin^2 x}\, dx = \int \frac{1}{\sin x}\frac{\cos x}{\sin x}\, dx = \int \csc x \cot x\, dx = -\csc x + C$$

Formula 8 in Table 5.2.1

Solution (b).

$$\int \frac{t^2 - 2t^4}{t^4}\, dt = \int \left(\frac{1}{t^2} - 2\right) dt = \int (t^{-2} - 2)\, dt$$

$$= \frac{t^{-1}}{-1} - 2t + C = -\frac{1}{t} - 2t + C$$

(4-2) Definite Integrals:

Theorem: If a function f is continuous on an interval [a,b], then f is integrable on [a,b].

The definite integral can be evaluated by finding any antiderivative of the integrand and then subtracting the value of this antiderivative at the lower limit of integration from its value at the upper limit of integration.

Although our evidence for this result assumed that f is nonnegative on $[a, b]$, this assumption is not essential.

5.6.1 THEOREM (*The Fundamental Theorem of Calculus, Part 1*). *If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then*

$$\int_a^b f(x)\, dx = F(b) - F(a) \tag{2}$$

Example 5 Table 5.2.1 will be helpful for the following computations.

$$\int_4^9 x^2\sqrt{x}\,dx = \int_4^9 x^{5/2}\,dx = \tfrac{2}{7}x^{7/2}\Big]_4^9 = \tfrac{2}{7}(2187-128) = \tfrac{4118}{7} = 588\tfrac{2}{7}$$

$$\int_0^{\pi/2} \frac{\sin x}{5}\,dx = -\frac{\cos x}{5}\Big]_0^{\pi/2} = -\frac{1}{5}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = -\frac{1}{5}[0-1] = \frac{1}{5}$$

$$\int_0^{\pi/3} \sec^2 x\,dx = \tan x\Big]_0^{\pi/3} = \tan\left(\frac{\pi}{3}\right) - \tan 0 = \sqrt{3} - 0 = \sqrt{3}$$

$$\int_{-\pi/4}^{\pi/4} \sec x \tan x\,dx = \sec x\Big]_{-\pi/4}^{\pi/4} = \sec\left(\frac{\pi}{4}\right) - \sec\left(-\frac{\pi}{4}\right) = \sqrt{2} - \sqrt{2} = 0$$

(4-3) Properties of definite Integrals:

(a) If a is in the domain of f, we define

$$\int_a^a f(x)\,dx = 0$$

(b) If f is integrable on $[a, b]$, then we define

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

c)

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

Example 6

$$\int_1^1 x^2\,dx = \frac{x^3}{3}\Big]_1^1 = \frac{1}{3} - \frac{1}{3} = 0$$

$$\int_4^0 x\,dx = \frac{x^2}{2}\Big]_4^0 = \frac{0}{2} - \frac{16}{2} = -8$$

The latter result is consistent with the result that would be obtained by first re
limits of integration in accordance with Definition 5.5.3(b):

$$\int_4^0 x\,dx = -\int_0^4 x\,dx = -\frac{x^2}{2}\Big]_0^4 = -\left[\frac{16}{2} - \frac{0}{2}\right] = -8$$

Example 7 Evaluate $\int_0^6 f(x)\,dx$ if

$$f(x) = \begin{cases} x^2, & x < 2 \\ 3x - 2, & x \ge 2 \end{cases}$$

Solution. From Theorem 5.5.5

$$\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = \int_0^2 x^2\,dx + \int_2^6 (3x-2)\,dx$$

$$= \frac{x^3}{3}\bigg]_0^2 + \left[\frac{3x^2}{2} - 2x\right]_2^6 = \left(\frac{8}{3} - 0\right) + (42 - 2) = \frac{128}{3}$$

Example 8 Evaluate $\int_{-1}^2 |x|\,dx$.

Solution. Since $|x| = x$ when $x \ge 0$ and $|x| = -x$ when $x \le 0$,

$$\int_{-1}^2 |x|\,dx = \int_{-1}^0 |x|\,dx + \int_0^2 |x|\,dx$$

$$= \int_{-1}^0 (-x)\,dx + \int_0^2 x\,dx$$

$$= -\frac{x^2}{2}\bigg]_{-1}^0 + \frac{x^2}{2}\bigg]_0^2 = \frac{1}{2} + 2 = \frac{5}{2}$$

EXAMPLE 1 Suppose that

$$\int_{-1}^1 f(x)\,dx = 5, \qquad \int_1^4 f(x)\,dx = -2, \qquad \int_{-1}^1 h(x)\,dx = 7.$$

1. $\int_4^1 f(x)\,dx = -\int_1^4 f(x)\,dx = -(-2) = 2$

2. $\int_{-1}^1 [2f(x) + 3h(x)]\,dx = 2\int_{-1}^1 f(x)\,dx + 3\int_{-1}^1 h(x)\,dx$

$= 2(5) + 3(7) = 31$

3. $\int_{-1}^4 f(x)\,dx = \int_{-1}^1 f(x)\,dx + \int_1^4 f(x)\,dx = 5 + (-2) = 3$